12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

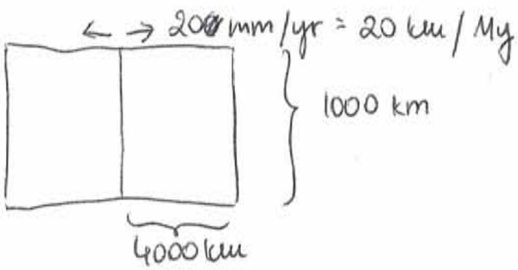
12.002 P Set 4 fall 2008 Solutions

① use the formula from the notes: $Q_s = \frac{K(T_{in}-T_s)}{\sqrt{\pi \kappa t}}$

$$t = \left(\frac{K(T_{in}-T_s)}{\sqrt{\pi\kappa}\, Q_s}\right)^2 \approx 170.6 \text{ My}$$

Kelvin assumed that Earth cooled uniformly by conduction and so that Q_s was decreasing through time according to the formula. He did not take into account that due to convection the temperature of the upper part of the mantle is kept higher and Q_s is higher than it would be without convection. He also didn't take into account the radioactive decoy.

②



$$20 \frac{km}{My} \cdot 200\,My = 4000\,km$$

a) $A = 1000 \cdot 4000 = 4\cdot 10^6\ km^2$

b) c) d) calculate the average heat flow from 0 to T age of lithosphere

$$Q_{avg} = \frac{1}{T}\int_0^T \frac{K(T_{in}-T_s)}{\sqrt{\pi\kappa t}}\,dt = \frac{1}{T}\frac{K(T_{in}-T_s)}{\sqrt{\pi\kappa}}\,2\sqrt{T} = \frac{2K(T_{in}-T_s)}{\sqrt{\pi\kappa T}}$$

then heat loss in W is equal to $Q = A \cdot Q_{avg}$ (↑ area)

see other sides for results for different T.

e) Q_{avg} we got are much higher than the Q_s we assumed in problem 1. It shows that convection increases Q_s.

③ To calculate subsidence use the formula from the notes: $\Delta l = \sqrt{\frac{\kappa t}{\pi}}\, 2\alpha (T_m - T_s)\frac{\rho_m}{\rho_m-\rho_w}$

Remember to multiply by $\frac{\rho_m}{\rho_m-\rho_w}$ as the space in the column is filled with water, which has non-negligible weight.

To calculate thickness use the same approach as for the last problem in p-set4.

use $\frac{T_b}{T_m} = 0.9923$ (T_b = 1290°C, T_m = 1300°C) $\Rightarrow$ $\mathrm{erf}(1.88) = 0.9923$

$$\frac{z}{2\sqrt{\kappa t}} = 1.88 \Rightarrow z = 1.88 \cdot 2 \cdot \sqrt{\kappa t}$$

→ see other side for results.

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% P1___________________________________________

q = 0.05;                                % W/m^2
Tin = 1300;                              % C
Ts = 0;                                  % C
K = 5;                                   % W/mK
k = 10^-6;                               % m^2/s

t = (K*(Tin-Ts)/(sqrt(pi*k)*q))^2;
p1_t_my = t/(60*60*24*365*10^6)

% P2___________________________________________

age = [200 10 50];                       % my
age_sec = age*60*60*24*365*10^6;         % s

p2_q_avg = 2*K*(Tin-Ts)./sqrt(pi*k*age_sec)

area = age*20*1000*10^6;                 % m^2
heat = area.*p2_qavg;                    % W
p2_heat_loss_W = heat

% P3___________________________________________

alpha = 3*10^-5;                         % 1/C
Tb = 1290;                               % C
rho_m = 3300;                            % kg/m^3
rho_w = 1000;                            % kg/m^3
t = [1 10 100];                          % my
t_sec = t*60*60*24*365*10^6;             % s

subsidence = sqrt(k.*t_sec/pi)*2*alpha*(Tin-Ts)*(rho_m/(rho_m-rho_w));
p3_subsidence_km = subsidence/1000

z = 1.88*2*sqrt(k*t_sec);
p3_thickness_km = z/1000
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p1_t_my =
  170.5808

(2)      b)          c)          d)
p2_q_avg =
    0.0924    0.4130    0.1847

p2_heat_loss_W =
   1.0e+11 *
    3.6941    0.8260    1.8471

(3)
p3_subsidence_km =
    0.3546    1.1213    3.5458

p3_thickness_km =
   21.1150   66.7715  211.1500
      1         10        100      My
```