12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

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Solution to Problem Set #1: Geochronology and the Age of the Solar System

1. Read the classic 1956 paper: Patterson, C., Age of meteorites and the Earth, *Geochim. Cosmochim. Acta*, 10, 230-237, 1956. This presents the first definitive radiometric age of the Earth and solar system.

(a) Starting with the rate expressions for radioactive decay of ^{238}U and ^{235}U, derive Patterson's equation (1) on page 231. What do you need to assume about the age and initial lead content of the two meteorites?

The rate expressions for radioactive decay of ^{235}U and ^{238}U are:

$$^{207}\mathrm{Pb} = \left(^{207}\mathrm{Pb}\right)_0 + {}^{235}\mathrm{U}\left(e^{\lambda_1 T} - 1\right) \quad \text{and} \quad ^{206}\mathrm{Pb} = \left(^{206}\mathrm{Pb}\right)_0 + {}^{238}\mathrm{U}\left(e^{\lambda_2 T} - 1\right)$$

where the ^{207}Pb, ^{206}Pb, ^{235}U and ^{238}U refer to the concentrations of these isotopes at time T, $(^{207}\mathrm{Pb})_0$ and $(^{206}\mathrm{Pb})_0$ are the initial concentrations of these isotopes, and λ_1 and λ_2 are the ^{235}U and ^{238}U decay constants, respectively. Dividing both sides of each equation by the concentration of ^{204}Pb (which is nonradiogenic) gives

$$\frac{^{207}\mathrm{Pb}}{^{204}\mathrm{Pb}} = \left(\frac{^{207}\mathrm{Pb}}{^{204}\mathrm{Pb}}\right)_0 + \frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\left(e^{\lambda_1 T} - 1\right) \quad \text{and} \quad \frac{^{206}\mathrm{Pb}}{^{204}\mathrm{Pb}} = \left(\frac{^{206}\mathrm{Pb}}{^{204}\mathrm{Pb}}\right)_0 + \frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\left(e^{\lambda_2 T} - 1\right)$$

Letting $R_1 = {}^{207}\mathrm{Pb}/{}^{204}\mathrm{Pb}$ and $R_2 = {}^{206}\mathrm{Pb}/{}^{204}\mathrm{Pb}$ this becomes

$$R_1 = \left(R_1\right)_0 + \frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\left(e^{\lambda_1 T} - 1\right) \quad \text{and} \quad R_2 = \left(R_2\right)_0 + \frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\left(e^{\lambda_2 T} - 1\right)$$

Take two meteorites a and b for which we have measured these isotopes at time T. The above equations for these two meteorites are:

$$R_{1a} = \left(R_{1a}\right)_0 + \left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_a\left(e^{\lambda_1 T_a} - 1\right) \quad \text{and} \quad R_{2a} = \left(R_{2a}\right)_0 + \left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_a\left(e^{\lambda_2 T_a} - 1\right)$$

$$R_{1b} = \left(R_{1b}\right)_0 + \left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b\left(e^{\lambda_1 T_b} - 1\right) \quad \text{and} \quad R_{2b} = \left(R_{2b}\right)_0 + \left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b\left(e^{\lambda_2 T_b} - 1\right)$$

Subtracting the lower equations from the upper equations gives:

$$R_{1a} - R_{1b} = (R_{1a})_0 - (R_{1b})_0 + \left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_a \left(e^{\lambda_1 T_a} - 1\right) - \left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b \left(e^{\lambda_1 T_b} - 1\right)$$

$$R_{2a} - R_{2b} = (R_{2a})_0 - (R_{2b})_0 + \left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_a \left(e^{\lambda_2 T_a} - 1\right) - \left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b \left(e^{\lambda_2 T_b} - 1\right)$$

Assuming the meteorites have the same age T and initially had the same lead isotopic ratios, these simplify to

$$R_{1a} - R_{1b} = \left[\left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_a - \left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b\right]\left(e^{\lambda_1 T} - 1\right)$$

$$R_{2a} - R_{2b} = \left[\left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_a - \left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b\right]\left(e^{\lambda_2 T} - 1\right)$$

Now, if $^{235}\mathrm{U}_a = c\ ^{235}\mathrm{U}_b$ for some constant c, then using $^{235}\mathrm{U}/^{238}\mathrm{U} = 1/k$ (that is, *assuming that the ratio of uranium isotopes is constant for all samples*), we see that $^{238}\mathrm{U}_a = c\ ^{238}\mathrm{U}_b$. Therefore the above equations become

$$R_{1a} - R_{1b} = \left[c\left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b - \left(\frac{^{235}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b\right]\left(e^{\lambda_1 T} - 1\right)$$
$$= \frac{(c-1)}{^{204}\mathrm{Pb}}\,^{235}\mathrm{U}_b\left(e^{\lambda_1 T} - 1\right)$$

$$R_{2a} - R_{2b} = \left[c\left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b - \left(\frac{^{238}\mathrm{U}}{^{204}\mathrm{Pb}}\right)_b\right]\left(e^{\lambda_2 T} - 1\right)$$
$$= \frac{(c-1)}{^{204}\mathrm{Pb}}\,^{238}\mathrm{U}_b\left(e^{\lambda_2 T} - 1\right)$$

Dividing these equations and substituting $k = {}^{238}\mathrm{U}/{}^{235}\mathrm{U}$ we obtain

$$\frac{R_{1a} - R_{1b}}{R_{2a} - R_{2b}} = \frac{\left(e^{\lambda_1 T} - 1\right)}{k\left(e^{\lambda_2 T} - 1\right)}$$

(b) If the solar system were now 6 B.y. old rather than 4.5 B.y. old, would the slope of the isochron be shallower or steeper than that in Patterson's Fig. 1? What if the solar system were now 3 B.y. old?

If the solar system were now 6 B.y. old (older than what was found in Fig. 1) then the slope of the isochron would be steeper, and if it had instead were 3 B.y. old (younger than what was found in Fig. 1), then the slope of the isochron would be shallower. This follows from plugging different values of T into Course Notes equation giving the slope of the isochron derived in class:

$$M = \frac{\left(e^{\lambda_1 T} - 1\right)}{k\left(e^{\lambda_2 T} - 1\right)}$$

In the upper figure at the end of the solutions you can see that this function is increasing.

(c) On Patterson's Fig. 1 (reproduced below), roughly sketch how the isochron of the meteorites would have appeared to a hypothetical observer about a billion years ago when the meteorite was only about three and a half billion years old. Is the isochron steeper or shallower than the present-day isochron? Can you reconcile this with your answer to (b)?

The isochron as it would have appeared to an observer several a couple years ago would have been steeper than the present-day isochron. The key to understanding this problem is that in the past, the $k = {}^{238}\text{U}/{}^{235}\text{U}$ ratio was smaller. You can see this by obtaining an expression for $k(T)$. Using

$$^{235}\text{U} = \left({}^{235}\text{U}\right)_0 e^{-\lambda_1 T} \quad \text{and} \quad {}^{238}\text{U} = \left({}^{238}\text{U}\right)_0 e^{-\lambda_2 T}$$

we see that

$$k = \frac{\left({}^{238}\text{U}\right)_0 e^{-\lambda_2 T}}{\left({}^{235}\text{U}\right)_0 e^{-\lambda_1 T}}$$

which shows that k decreases monotonically with time because $\lambda_1 > \lambda_2$. Although k is constant spatially in nature, it is not constant in time! The equation for the slope of the line (see answer to part (b)) therefore implies that the isochron slope in the past would have been steeper, as it is as follows:

$$M = \frac{\left({}^{235}\text{U}\right)_0 e^{-\lambda_2 T}}{\left({}^{238}\text{U}\right)_0 e^{-\lambda_1 T}} \frac{\left(e^{\lambda_1 T} - 1\right)}{\left(e^{\lambda_2 T} - 1\right)} = \frac{\left({}^{235}\text{U}\right)_0}{\left({}^{238}\text{U}\right)_0} \frac{\left(1 - e^{-\lambda_1 T}\right)}{\left(1 - e^{-\lambda_2 T}\right)}$$

You can see on the lower figure at the end of the solutions that this function is decreasing with time.

This at first glance seems to contradict your answer to part (b) that older samples have steeper isochrons. In fact, there is no contradiction because the latter is only true for samples observed *today*. The isochrons of samples shallow as the samples age, but older samples start out their existence with steeper isochrons such that their present-day isochrons are still steeper than younger samples.

(d) The data for the Canyon Diablo meteorite were collected by measuring the Pb isotopic composition in the mineral troilite. What is the chemical formula of this mineral? Would you expect to find more Pb or U in troilite? How does this help us to constrain the initial Pb isotopic composition of the Earth?

The chemical formula is FeS. Pb is chalcophile (sulfur-loving) element while U is lithophile (silicate-loving): Pb is present in the solar system naturally as Pb^{+2} ions (like Fe in troilite, which is Fe^{+2}) while U is present naturally as U^{+4} and U^{+6} (much different from Fe in troilite). As a result, Pb but not U readily substitutes for Fe, and therefore FeS initially incorporates Pb and not U. This meant that very little radiogenic lead accrued in the meteorite over the history of the solar system and the Pb content observed today is very similar to the initial Pb content.

(e) Use the data in Patterson's Table 1 to determine the age of the solar system. Do this by recreating his Figure 1: plot the Pb isotopic composition of the five meteorites and determine the slope of the best fit line that passes through the Canyon Diablo meteorite. Note that because equation (1) is transcendental, you will need to calculate its right side iteratively in order to solve for time.

See attached Excel sheet.

(f) Based on these results, what is the minimum age for the formation of the crust of asteroid 4 Vesta? What is the minimum age for the formation of the core of the asteroid that produced Meteor Crater in Arizona?

Patterson showed that the crystallization age of Nuevo Laredo is 4.55-4.56 Ga. Nuevo Laredo is a eucrite and is thought to be from the differentiated asteroid 4 Vesta. Therefore 4.55 Ga is a minimum age for the formation of Vesta's crust.

The meteoroid which formed Meteor Crater was Canyon Diablo's parent body. Canyon Diablo is an iron meteorite and is thought to be from its parent body's core. Therefore its 4.55 Ga is also a minimum age for the formation this core.

(g) Now, going beyond Patterson to results from the modern era, what is the name of given to the oldest known solar system solids? What is their precise age (to within 1 million years)? Give a first order explanation for their composition based on the expected condensation sequence for the solar nebula.

The oldest known solar system solids are calcium aluminum inclusions (CAIs). The were dated precisely in 2002 by Y. Amelin to be 4567 Ma. A first order explanation for their composition is that Lewis' geochemical model for condensation of the solar nebula predicts that the minerals with highest vaporization points will condense first. You can see from the handouts and dP&L Fig. 128b,c that Ca- and Al-rich minerals have some of the highest condensation temperatures (are very *refractory*).

Graphs to b) and c).